



Kentucky Department of

***Our
Children,***

***Our
Commonwealth***

Education



Standards for Mathematical Practice

Webcast
May 9, 2019

Essential Questions:

- ▶ What are the Standards for Mathematical Practice?
- ▶ Why are the Standards for Mathematical Practice important?
- ▶ Where are the Standards for Mathematical Practice?
- ▶ How can educators get additional support with planning instruction that will engage the Standards for Mathematical Practice?



What are the Standards for Mathematical Practice?

- ▶ The **Standards for Mathematical Content** are a balanced combination of conceptual understanding, procedural skill/fluency and application.
- ▶ The **Standards for Mathematical Practice** describe ways in which developing student practitioners of mathematics should increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.



Standards for Mathematical Practice

- ▶ MP.1 Make sense of problems and persevere in solving them.
- ▶ MP.2 Reason abstractly and quantitatively.
- ▶ MP.3 Construct viable arguments and critique the reasoning of others.
- ▶ MP.4 Model with mathematics.
- ▶ MP.5 Use appropriate tools strategically.
- ▶ MP.6 Attend to precision.
- ▶ MP.7 Look for and make use of structure.
- ▶ MP.8 Look for and express regularity in repeated reasoning.



Why are the Standards for Mathematical Practice so important?

- ▶ Students who lack understanding of a topic may rely on procedures too heavily and may be less likely to consider similar problems, represent problems coherently, justify conclusions, apply mathematics to practical situations, use technology mindfully, and explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.



Where are the Standards for Mathematical Practice?




Statistics and Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Develop understanding of statistical variability.	

Standards	Clarifications
KY.6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. MP.1, MP.3, MP.6	For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates a variety of values with associated variability in students’ ages. Coherence KY.5.MD.2→KY.6.SP.1→KY.7.SP.1
KY.6.SP.2 Understand that a set of numerical data collected to answer a statistical question has a distribution which can be described by its center, spread and overall shape. MP.2, MP.6, MP.7	Students distinguish between graphical representations which are skewed or approximately symmetric; use a measure of center to describe a set of data. Coherence KY.5.MD.2→KY.6.SP.2→KY.7.SP.3
KY.6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number to describe a typical value, while a measure of variation describes how the values in the distribution vary. MP.2, MP.5, MP.6	Emphasis is on the sensitivity of measures of center to changes in the data, such as mean is generally much more likely to be pulled towards an extreme value than the median. Additionally, measures of variation (range, interquartile range) describe the data by giving a sense of the spread of data points. Coherence KY.6.SP.3→KY.7.SP.4

Attending to the Standards for Mathematical Practice

Students recognize a question such as “What did I eat for breakfast?” is not a statistical question, whereas “What is the most popular breakfast in my school?” will elicit data they can measure precisely (**MP.6**) and draw conclusions based on that data (**MP.3**). After collecting data, by creating a distribution of that data, students recognize data generally follows a structure and can be described in terms of that structure (**MP.7**). By accurately calculating the mean (or any other statistical measure), students are now more precise in describing data, going from, for example, describe the rainfall for the month as “about average” to “the rainfall this month is slightly higher than the mean of the last 10 years and within the interquartile range for that data.” (**MP.6**)

How can educators get additional support with planning instruction that will engage the Standards for Mathematical Practice?

- ▶ *Getting to Know the Kentucky Academic Standards for Mathematics* Module:
 - Section A: Revision Process Overview
 - Section B: Understanding the Architecture
 - ▶  ● Section C: A Closer Look at the Standards for Mathematical Practice
 - Section D: A Closer Look at the Standards for Mathematical Content
 - Section E: Spotlight: Clarifications & Coherence
 - Section F: Spotlight: Front Matter & Appendix A
 - Section G: Wrap Up & Next Steps



Section 1C: A Closer Look: Standards for Mathematical Practice



- ▶ Essential idea: Educators around the state have varying levels of experience with designing and implementing instruction that attends to the practice standards. The rollout of the *KAS for Mathematics* presents a great opportunity for educators around the state to develop a shared understanding of how to engage students in the practices.
- ▶ Provides a focused learning experience around the Standards of Mathematical Practice including:
 - Task: Attending to the SMPs
 - Task: Sample Task Match-up
 - Resource: Engaging the SMPs: Look fors & Question Stems
 - Optional Extension: Reflection on Current Instructional Choices

Discussion within module:

MP.1 Make sense of problems & persevere in solving them.

Possible Teacher Actions:

- Providing rich problems aligned to the standards.
- Providing appropriate time for students to engage in the productive struggle of problem solving.
- Providing opportunities for students to solve problems that have multiple solutions.

Possible Student Actions:

- Working and reading rich problems carefully.
- Analyzing information.
- Drawing pictures, diagrams, tables, or using objects to make sense of the problem.
- Discussing the meaning of the problem with classmates.
- Trying out potential solution paths and making changes as needed.
- Checking answers and making sure solutions are reasonable and make sense.
- Exploring other ways to solve problems.
- Persisting in efforts to solve challenging problems, even after reaching a point of frustration.
- Relating current situations to concepts or skills previously learned and connect mathematical ideas to one another.



Kindergarten Task: Attending to the SMPs



Counting and Cardinality	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Know number names and the count sequence.	
Standards	Clarifications
KY.K.CC.1 Count a. Count to 100 by ones and by tens. b. Count backwards from 30 by ones. MP.7, MP.8	Students verbally count forward by ones (1,2,3,4. . .) to 100 Students verbally count forward by tens (10, 20, 30. . .) to 100. Students verbally count backwards by ones (30, 29, 28, 27. . .) from 30. Coherence KY.K.CC.1→KY.1.NBT.1
KY.K.CC.2 Count forward beginning from a given number within the known sequence within 100 (instead of having to begin at 1). MP.7	Students verbally count forward starting at a number other than one (58, 59, 60, 61, 62. . .) within 100. Coherence KY.K.CC.2→KY.1.NBT.1
KY.K.CC.3 Represent numbers. a. Write numbers from 0 to 20. b. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). MP.2, MP.7, MP.8	Students write all numerals in the range of 0-20 (1, 2, 3, 4, 5...) When students are given a written numeral, represent with objects within 20 (4... ★ ★ ★ ★). Coherence KY.K.CC.3→KY.1.NBT.1
Attending to the Standards for Mathematical Practice	
Students notice repetition inherent in the counting sequence as they count to one hundred by ones and tens. For example, students notice “seven” follows “six,” and “twenty-seven” follows “twenty-six” (). They describe how this pattern exists into new decade families. For example, thirty-seven follows thirty-six and so on. Students use this general pattern about how numbers are structured to count forward from any given number within the range of 0-100 (counting on) without the benefit of starting at “one” (). When counting objects within the range of 0-20, they understand they can communicate this total using words, for example “ten” and the numeral 10. ()	

Kindergarten Task: Sample Task Match Up

Module 1: Section 1C: A Closer Look at the Standards for Mathematical Practice:

Kindergarten Sample Tasks	Kindergarten Sample Tasks
<p style="text-align: center;">Task A:</p> <p>Materials:</p> <ul style="list-style-type: none"> * One of the student's shoes to use to compare to other items. * A bin of seven to ten commonly used classroom items, such as a glue bottle, a pair of scissors and a crayon, that are similar in size but distinctly longer or shorter than a students' shoe. * Sheets of paper, folded in half with the words 'longer' and 'shorter' written, in 2 different colors, at the top of each side. <div data-bbox="201 725 907 925" style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border-right: 1px solid black; padding: 5px; text-align: center; color: #00AEEF; font-weight: bold;">Longer</div> <div style="padding: 5px; text-align: center; color: #C00000; font-weight: bold;">Shorter</div> </div> </div> <p>Setup: All students have the prepared sheet of paper and a pencil.</p> <p>Action: The students begin by removing their shoe; this is their 'measuring item'. Then they select an item from the bin to measure against their 'measuring item'. They directly compare it by holding it against their item and decide if it is longer or shorter than their shoe. The students then draw a picture of it on the correct side of the longer/shorter sheet depending on how it measured up. They continue to compare items to measure against their shoe until they have 2-3 drawings on each side of their sheet.</p>	<p style="text-align: center;">Task B:</p> <p>Materials</p> <p>Long, skinny objects to compare; for example:</p> <ul style="list-style-type: none"> • a pair of scissors • a crayon • a glue stick • a long, skinny wooden block from the classroom block set • a marker <p>Actions</p> <p>The teacher will pre-select a group of classroom objects for the students to use. Each student will choose two objects to compare and they will lay them next to each other and compare which is longer. The teacher may need to show students that they need to make sure the starting ends are correctly lined up, like this:</p> <div data-bbox="1009 939 1472 1002" style="margin: 10px 0;"> </div> <p>Not like this this:</p> <div data-bbox="1009 1059 1533 1122" style="margin: 10px 0;"> </div> <p>The teacher can have the students record their findings in one of two ways:</p> <ul style="list-style-type: none"> * Students can trace both objects on a black piece of white paper. The students can label their drawings depending on their literacy skills and then circle the longer object. * Students can use the attached blackline master. This requires higher level skills as students must decide which object is the longer and shorter object, conserve that information in their brain and then write

Module 1: Section 1C: A Closer Look at the Standards for Mathematical Practice: Kindergarten Sample Tasks

Participant Guide

Directions: Match each task to the SMP targeted by the author of the task. While some tasks may connect to more than one SMP, there is one task that most closely aligns with each of the SMPs. Thus, each SMP will have one task to match and each task will only be used once.

Note: The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, only one practice connection will be discussed in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

Standards of Mathematical Practice (SMP)	Standards of Mathematical Practice (SMP)
MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the practice standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from Illustrative Mathematics.

Facilitator's Guide

Throughout facilitation of this activity it will be important to remind participants:

- Use the cluster level narratives to better understand what attending to the mathematical practices might look like in the classroom.
- Emphasize to participants the statement at the end of each cluster within the *KAS for Mathematics*, "The identified mathematical practices, coherence connections, and clarifications are possible suggestions; however, they are not the only pathways."

Standards of Mathematical Practice (SMP)	Standards of Mathematical Practice (SMP)
<p>MP.1. Make sense of problems and persevere in solving them.</p> <p style="text-align: center;">Task D:</p> <p>Kindergartners are exposed to multiple problems through the story, <i>The Napping House</i>. As students listen to the story they use counters on a ten frame to keep track of each additional person/animal who gets in the bed until the flea bites the mouse. Then the story changes to subtraction as people/animals start to leave the bed. Throughout this guided task, students are introduced to the processes of problem-solving in a non-threatening way. They are able to unpack the parameters of the problem by manipulating the counters one at a time. This allows them to make sense of the actions occurring in the story. These concrete objects help them to conceptualize and solve each problem as posed in the story. The teacher can guide this conceptualization by stopping after each action and asking questions such as, "What just happened in the story?" "How are we going to show that on our ten frames?" "How many are in the bed now?" and "How do you know how many are in the bed?"</p>	<p>MP.5. Use appropriate tools strategically.</p> <p style="text-align: center;">Task A:</p> <p>During this exploration, young learners investigate the attribute of length by directly comparing two objects and deciding which object is shorter and which is longer (one of the objects is the student's shoe). Kindergartners will easily directly compare lengths in simple situations. In this case, the shoe becomes the measuring tool because it is consistently used for all comparisons the child makes. As students become proficient in this practice, they will be able to consider a tool's usefulness and consider its strengths and limitations, as well as know how to use it appropriately. Since this may be a new experience for kindergartners, there will be learning involved as to how to position the two objects to accurately compare their lengths.</p> <p>The necessity of aligning endpoints can be explicitly addressed and reinforced throughout this task (MP.6). The opportunity for conversation often occurs in comparison situations ("The teacher's</p>

Please note that inclusion of these sample tasks does not represent that this task is endorsed by or rejected by the Kentucky Department of Education. Inclusion of these tasks was for the sole purpose of allowing participants the opportunity to investigate the practice standards within the *Kentucky Academic Standards for Mathematics* more closely. All tasks were selected from Illustrative Mathematics.

Engaging the SMPs: Look-fors & Question Stems

Engaging the SMPs: Look-fors & Question stems

Standard for Mathematical Practice 1: Make sense of problems and persevere in solving them.

Possible Student Actions: Students are...	Possible Teacher Actions: Teachers are...	Possible Questions to Promote: Teachers ask...
<ul style="list-style-type: none"> <input type="checkbox"/> Working and reading rich problems carefully. <input type="checkbox"/> Analyzing information (givens, constraints, relationships, goals). <input type="checkbox"/> Drawing pictures, diagrams, tables, or using objects to make sense of the problem. <input type="checkbox"/> Discussing the meaning of the problem with classmates. <input type="checkbox"/> Making choices about which solution path to take. <input type="checkbox"/> Trying out potential solution paths and making changes as needed. <input type="checkbox"/> Checking answers and making sure solutions are reasonable and make sense. <input type="checkbox"/> Exploring other ways to solve problems. <input type="checkbox"/> Persisting in efforts to solve challenging problems, even after reaching a point of frustration. <input type="checkbox"/> Relating current situations to concepts or skills previously learned and connect mathematical ideas to one another. <p>Comments:</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Providing rich problems aligned to the standards. <input type="checkbox"/> Providing appropriate time for students to engage in the productive struggle of problem solving. <input type="checkbox"/> Providing opportunities for students to solve problems that have multiple solutions. <p>Comments:</p>	<ul style="list-style-type: none"> <input type="checkbox"/> What information do you have? <input type="checkbox"/> What do you need to find out? <input type="checkbox"/> What do you think the answer might be? <input type="checkbox"/> Can you draw a picture? <input type="checkbox"/> How could you make this problem easier to solve? <input type="checkbox"/> Have you compared your work with anyone else? <input type="checkbox"/> How is ___'s way of solving the problem like/different from yours? <input type="checkbox"/> Does your plan make sense? Why or why not? <input type="checkbox"/> What tools/manipulatives might help you? <input type="checkbox"/> What are you having trouble with? <input type="checkbox"/> How can you check this? <input type="checkbox"/> What do you think about what ___ said? Do you agree? Why or why not? <input type="checkbox"/> How might you use one of your previous problems to help you begin? <input type="checkbox"/> What are some other problem that are similar to this one? <p>Comments:</p>



June Learning Lab Session



- ▶ In this session, participants will take notice of the intentional emphasis the writers placed on the Standards of Mathematical Practice (SMP) within the *KAS for Mathematics* and will recognize the impact that will need to have on planning and implementing classroom instruction moving forward.
- ▶ Participants will come away with an understanding of the following questions below:
 - Why are the standards for mathematical practice so important?
 - How can having the Attending to the Mathematical Practices component within each cluster of the *KAS for Mathematics* provide direction to teachers when designing cohesive instruction?
 - What would be the value in reflecting upon current lessons, units, etc. to determine whether the instruction will address each of the eight SMPs?

5 Things to Know Before You Go

- ▶ Choosing and supporting PL facilitators
- ▶ June Conference
- ▶ kystandards.org
- ▶ Remember to contact your cooperative for support
- ▶ Next week's webcast:
Architecture of the Newly Revised Social Studies Standards: The *Kentucky Academic Standards for Social Studies*





Kentucky Department of

Our Children,

Our Commonwealth

Education